Unified Engineering Fall 2007
Signals & Systems Problem 1

Math review (10 points)

In this question, you will review some math skills that we will use regularly during the Signals & Systems lectures.

a) Complex numbers (Reading: ow page 71; Review: 18.01, 18.03 notes)
Express each of the following complex numbers in polar form, $z = re^{j\theta}$. Plot each one as a vector in the complex plane. On your plot, indicate the magnitude r and angle θ of each vector.

(i) $z_1 = 1 + j\sqrt{3}$

(ii) $z_2 = \frac{e^{j\pi/3}-1}{1+j\sqrt{3}}$

(ii) $z_3 = (1 + j)^5$

b) Function manipulation (Reading: ow 1.1, 1.2, 1.4).
(from P1.21 in ow) A signal $x(t)$ is shown in the figure below. Sketch and label carefully each of the following signals. Note that in our notation below, $\sigma(t)$ is the unit step function and $\delta(t)$ is the unit impulse function.

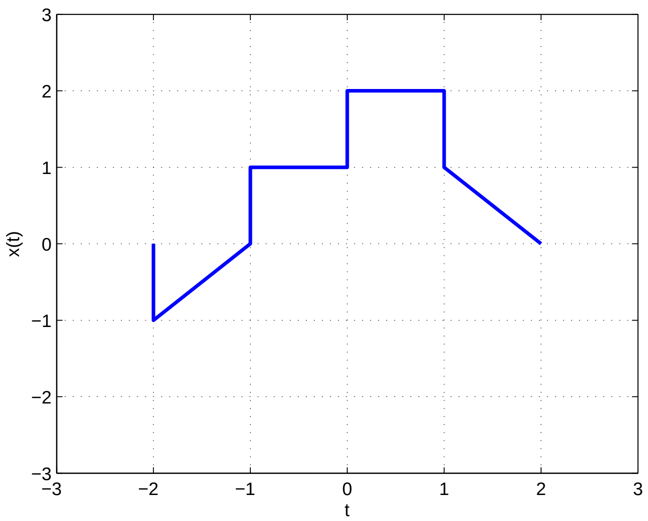

Figure 1: Figure P1.21 (Oppenheim and Willsky).

(i) $x(t - 1)$

(ii) $x(2 - t)$

(iii) $x(2t + 1)$

(iv) $[x(t) + x(-t)]\,\sigma(t)$

(v) $x(t)\left[\delta(t+\frac{3}{2}) - \delta(t-\frac{3}{2})\right]$

c) Linear differential equations (Review: 18.03 notes)

(i) Compute the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

(ii) Using your results from (i), solve the following initial value problem to compute $x_1(t)$ and $x_2(t)$, $t \geq 0$:

$$\begin{aligned} \frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \\ x_1(0) &= 1 \\ x_2(0) &= 0 \end{aligned}$$